Backstepping Based Nonlinear Ammonia Surface Coverage Ratios Control for Diesel Engine Selective Catalytic Reduction Systems

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Abstract—This paper presents a selective catalytic reduction (SCR) ammonia surface coverage ratio control design based on a two cell SCR model. It has been studied that the real SCR dynamics can be sufficiently presented by two lumped parameter SCR models (cells). Also a two cell model provides more control degree of freedom for NOx and ammonia slip reduction. However, connecting two cells together greatly increase the system complexity, and controlling ammonia surface coverage ratios in two cells by one control input is challenging. To solve these problems, a backstepping-based controller is purposed to regulate the ammonia coverage ratio of the upstream cell to a desired value in order to maximize the NOx conversion efficiency, at the same time, the coverage ratio of the downstream cell is controlled to be under a prescribed value such that any over limit NH3 slip can be prevented. Simulations were conducted based on the first phase of a FTP75 cycle, results showed the purposed controller can regulate the ammonia coverage ratio in each cell very and the control requirements are satisfied.

I. INTRODUCTION

Diesel engines are noted for its better fuel economics and power density comparing to gasoline engines, and have been used by majority of heavy duty vehicles for a long time. However, it was not popularly adopted by light duty cars because it can essentially generate more pollutant emissions than a gasoline engine. Recently, diesel engines regained much attention in both heavy duty and light duty automotive areas due to many improved technologies which can significantly reduce the emissions. Among the technologies, selective catalytic reduction (SCR) is one of the most promising techniques that can decrease the diesel engine NOx emission to satisfy the US 2010 regulations. SCR utilizes ammonia as the reductant to reduce NOx by catalytic reactions inside a reactor. However, a major drawback of SCR is it can generate unpleasant odor if ammonia is over-dosed and more than acceptable amount are slipped out.

To solve this problem, special attention has to be paid on the NH3 dosing control. As mentioned before, over-dosing can cause ammonia slip and thus generate unpleasant smell and waste, but insufficient ammonia injection will result in low NOx reducing efficiency, which can cause higher NOx emission such that the engine may not be able to satisfy the emission regulations.

It was purposed in [2] and demonstrated in [3] that a possible way to minimize the NOx emission and limit the ammonia slip is via controlling the ammonia surface coverage inside the SCR reactor to an optimal value. The ammonia surface coverage ratio $\theta_{NH3}$ is defined as:

$$\theta_{NH3} = \frac{1 - M_{NH3}^*}{\theta}.$$  (1)

Even though there is no sensor available currently to directly measure this value, [4] has purposed a possible method to measure this value by lab environments, and observers based on available sensors have been purposed in [5], [6], and [7]. We believe ammonia surface coverage ratio regulation is a proper way of SCR control. In this paper, we follow this concept and select ammonia surface coverage as the control objective.

However, SCR control itself is not an easy task. The chemical reactions inside the reactor make the controlled plant very nonlinear and the changing engine exhaust conditions result in a time varying system. Besides, the reactor is generally large enough such that it cannot be modeled by a simple continuous stirred tank reactor (CSTR) model (nominated as a SCR cell in the rest of the paper) that assumes all states inside the reactor is homogeneous. Authors of [1] pointed out that the SCR dynamics can be accurately captured by at least two CSTR models. For this reason, a multiple cells model is necessary, but combination of cells eventually increased the complexities of the control problem significantly.

In [8], the authors purposed a controller utilizing backstepping concept to control the surface coverage ratio of a SCR model with multiple cells. The control objective of the study was to regulate the surface coverage of the last cell such that the NH3 slip can be controller under a limitation. It was demonstrated by simulations that the backstepping based strategy can regulate the objective fairly well. In this paper, we utilize a SCR model with two cells, and extend the control objectives from controlling the coverage ratio of the last cell to coverage ratios of the two cells. It was pointed out in [4] that, in principle, minimization of NOx and NH3 emissions can be achieved via controlling the coverage ratio of the upstream portion to a higher value and a lower value in the downstream portion. Based on this knowledge, the
backstepping based controller was designed to regulate the upstream cell to a desired value to convert most emission NOx 
emission to N2, at the same time, bring the coverage of the 
downstream cell to a level lower than a prescribed value in 
order to adsorbed ammonia slip from the upstream cell. 

The rest of this paper is organized as follow. A brief 
introduction of the SCR model is presented in the following 
section. After that, the SCR controller design is described 
and analyzed, followed by simulation results based on the first 
phase of a FTP75 cycle. Conclusive remarks are summarized 
at the end.

II. SELECTIVE CATALYTIC REDUCTION

A. SCR Operation Principles

The SCR NOx reduction mainly includes three processes. 
In the first part, the injected urea is completely transformed 
to ammonia, NH3, the reductant [9]. The NH3 inside the reactor 
is then adsorbed on the catalyst surface. The adsorbed NH3 
can then catalytically react with NOx and convert them to 
nitrogen. The dominant chemical reactions and the 
corresponding reaction rates are briefly explained below.

1) NH3 Adsorption/Desorption

The NH3 entered the SCR reactor can be adsorbed on the 
SCR substrate. The adsorbed NH3 can also be desorbed 
from the substrate as shown in the following reaction equation.

\[ \text{NH}_3 + \theta \rightarrow \text{NH}_3^* \]  

The rate of the NH3 adsorption/desorption can be expressed 
by the following equations [10].

\[ r_{ads} = k_{ads} \exp \left( -\frac{E_{ads}}{RT} \right) C_{NH3} \left( 1 - \theta_{NH3} \right) \]  

\[ r_{des} = k_{des} \exp \left[ -\frac{E_{des}}{RT} \theta_{NH3} \right] \]  

Also, at temperature higher than 450 deg C or so, the 
adsorbed NH3 can be oxidized to NO by the following 
reaction.

\[ \text{NH}_3^* + 1.25O_2 \rightarrow NO + 1.5H_2O \]  

\[ r_{oxi} = k_{oxi} \exp \left( -\frac{E_{oxi}}{RT} \right) \theta_{NH3} \]  

2) NOx Reduction

The adsorbed NH3 can then catalytically react with NOx to 
become nitrogen according to the following reactions.

\[ 4\text{NH}_3^* + 4\text{NO} + O_2 \rightarrow 4\text{N}_2 + 6\text{H}_2\text{O} \]  

\[ 2\text{NH}_3^* + \text{NO} + \text{NO}_2 \rightarrow 2\text{N}_2 + 3\text{H}_2\text{O} \]  

\[ 4\text{NH}_3^* + 3\text{NO}_2 \rightarrow 3.5\text{N}_2 + 6\text{H}_2\text{O} \]  

Because more than 90% of the diesel exhaust gas NOx is 
usually composed of NO [11], reaction in Eq. (12) is 
considered the dominant reaction in NOx reduction. The 
reaction rate is described below [5].

\[ r_{red} = k_{red} \exp \left[ -\frac{E_{red}}{RT} \right] C_{NO} \theta_{NH3} \]  

B. SCR Model

Based on the molar balance, the main SCR dynamics can be 
described by the following equations [10]:

\[ \dot{C}_{NO} = \theta(r_{oxi} - r_{ads}) \]  

\[ \dot{\theta}_{NH3} = r_{ads} - r_{des} - r_{red} - r_{oxi} \]  

\[ \dot{C}_{NH3} = r_{des} - r_{ads} \]  

Such dynamic equations can be used to develop the SCR 
model using Continuous Stirred Tank Reactor (CSTR) 
approach or mass conservation law. Many SCR models for 
vehicle applications have been proposed in the literature [6], 
[10], [12], and [13]. In this paper, we adopted the model 
presented in [10]. The nonlinear model in state-space form is 
shown in Eq. (14).

\[ \begin{align*} 
\dot{C}_{NO} & = -C_{NO} \left( \theta r_{red} \theta_{NH3} + \frac{F}{\nu} \right) + r_{NH3,oxi} \theta_{NH3} \\
\dot{\theta}_{NH3} & = -\theta_{NH3} \left( r_{ads} C_{NH3} + r_{des} + r_{red} C_{NO} + r_{oxi} \right) + r_{ads} C_{NH3} + r_{des} C_{NH3} \\
\dot{C}_{NH3} & = \theta_{NH3} \left( r_{ads} (1 - \theta_{NH3}) + \frac{F}{\nu} \right) + r_{des} \theta_{NH3} \\
\end{align*} \]  

However, the CSTR model assumes all the states are 
uniform throughout the SCR reactor, which may not be valid 
especially for large SCR reactors for medium- and heavy-duty 
diesel engine applications. For example, concentration of NO 
must be lower at downstream side than that at the upstream 
side of the reactor. The authors in [1] pointed out that to 
better capture the SCR dynamics, a SCR reactor can be 
divided into multiple cells, and the minimum number of SCR 
cells to sufficiently represent a real SCR converter is two. The 
SCR cell here means a single CSTR model as presented in Eq. 
(15). Following this physical insight, the SCR model to be 
studied in this paper is a two-cell SCR model which can be 
represented by Eq. (15) and Figure 1.

\[ \begin{align*} 
\dot{C}_{NO,i} & = -C_{NO,i} \left( \theta_{red,i} \theta_{NH3,i} + \frac{F}{\nu} \right) + r_{NH3,oxi,i} \theta_{NH3,i} \\
\dot{\theta}_{NH3,i} & = -\theta_{NH3,i} \left( r_{ads,i} C_{NH3,i} + r_{des,i} + r_{red,i} C_{NO,i} + r_{oxi,i} \right) + r_{ads,i} C_{NH3,i} + r_{des,i} C_{NH3,i} \\
\dot{C}_{NH3,i} & = \theta_{NH3,i} \left( r_{ads,i} (1 - \theta_{NH3,i}) + \frac{F}{\nu} \right) + r_{des,i} \theta_{NH3,i} \\
\end{align*} \]  

\( i = 1, 2, \) 

where C_{NH3,3} and C_{NO,31} are the controlled NH3 concentration 
and engine exhaust NO concentration into the SCR reactor, 
and C_{NH3,5} and C_{NO,1} are the tailpipe NH3 and NO concentrations.
Figure 1. SCR model with multiple cells

III. SCR CONTROLLER DESIGN

It was proposed in [4] that, to minimize the NOx and NH3 emissions, the ammonia coverage ratio in the upstream portion of the reactor should be higher and lower in the downstream portion. According to Eq. (10), higher coverage ratio can always achieve a higher NOx conversion efficiency. But an over-large ratio can induce to high NH3 slip and cause unpleasant odor as can be seen in Eq. (4), such that, according to the reaction in Eq.(3), lower coverage ratio at the downstream part is desired to adsorb extra ammonia before it leave. Theoretically, it is possible to find an optimal coverage ratio profile along the axial length of the reactor. By controlling the coverage ratio to this desired profile, the NOx conversion and NH3 slip can be balanced. But, practically, it is hard to be accomplished because controlling coverage ratio to a “profile” requires infinite number of cell which is impossible in reality. Besides, is hard to achieve a perfect tracking of the optimal profile because engine exhaust can change rapidly with time which causes large and unpredictable disturbances. Any tracking error in this case can cause extra ammonia slip and thus generate stench from engine exhaust.

To solve these problems, we propose a control strategy of dividing the SCR reactor into two cells, which is sufficient to represent the real SCR dynamics. The upstream cell is served as the NOx conversion cell, which is to be controlled to an optimal value and reduces most NOx. The optimal value can be obtained from experimental data, and it is the minimum value which can guarantee most NOx to be converted to N2 in this cell such that the exhaust NOx concentration will not reach the regulation. And the downstream cell is served as an ammonia reservoir cell which holds a lower coverage ratio and is mainly used to absorb NH3 slip from the upstream NOx conversion cell.

To achieve the above control objectives, we design an ammonia surface coverage ratios controller based on backstepping concept. The goals of the controller is to control the NH3 coverage ratio of the upstream cell, the second cell or the NOx conversion cell, to the optimal value, with the constraint that the coverage ratio of the downstream cell, the first cell or the ammonia reservoir cell, does not exceed a prescribed limit.

Considering the control input as $C_{NH3,1}$ and controlled output as $\theta_{NH3,1}$ and $\theta_{NH3,2}$, the control problem based on the model in Eq. (15) is formulated below. The objective of the controller is to regulate $\theta_{NH3,2}$ to the optimal value $\theta^*_2$ and, at the same time, keep $\theta_{NH3,1}$ under the prescribed value $\theta^*_1$.

$$\dot{\theta}_{NH3,1} = -C_{NH3,1}
\left[\theta_{1r_{ads,1}}(1 - \theta_{NH3,1}) + \frac{F}{v_1}\right]
+ \theta_{1r_{des,1}}\theta_{NH3,1} + \frac{F}{v_1}C_{NH3,2},$$

$$\dot{C}_{NH3,2} = -C_{NH3,2}
\left[\theta_{2r_{ads,2}}(1 - \theta_{NH3,2}) + \frac{F}{v_2}\right]
+ \theta_{2r_{des,2}}\theta_{NH3,2} + \frac{F}{v_2}C_{NH3,3},$$

$$\theta_{NH3,2} = -\theta_{NH3,2}(r_{ads,2}C_{NH3,2} + r_{des,2} + r_{red,2}C_{NO,2}
+ r_{ox,2}L_2) + r_{ads,2}C_{NH3,2}.$$
Proof:
The proof is based on the concept of backstepping. Two cases will be considered in the proof: \( x_1 > \theta_1^* \) and \( x_1 \leq \theta_1^* \). In the first case, since \( x_1 \leq \theta_1^* \) is the constraint that has to be satisfied, we will prove that \( x_1 \) can converge to \( \theta_1^* \) disregarding of \( x_4 \). In the second case, since the constraint is satisfied, we want to prove that \( x_4 \) can then converge to the optimal value \( \theta_2^* \).

1) Case I: \( x_1 > \theta_1^* \)

Considering the first equation in Eq. (17) as the first dynamic system of backstepping, we want to prove that \( x_1 \) can converge to the optimal value \( \theta^* \) by the control of \( x_2 \). The Lyapunov function candidate \( V_1 \) is defined as:

\[
V_1(\bar{x}_1, t) = \frac{1}{2} \bar{x}_1^2 > 0, \tag{32}
\]

where \( \bar{x}_1 = (x_1 - \theta_1^*) \) and \( 0 < \theta_1^* \leq 1 \). The time derivative of the Lyapunov function is:

\[
\dot{V}_1(\bar{x}_1, t) = \bar{x}_1 \dot{x}_1 = \bar{x}_1 f_1(\bar{x}_1 + \theta_1^*, \delta(t)) + \bar{x}_1 g_1(\bar{x}_1 + \theta_1^*, \delta(t)) x_2. \tag{33}
\]

Combining Eq. (33) and Eq. (16):

\[
\dot{V}_1(\bar{x}_1, t) = -r_{des,1}(t) + r_{red,1}(t) c_{N0,1}(t) + r_{ox1}(t) \bar{x}_1(\bar{x}_1 + \theta_1^*) + r_{ads,1}(t) [1 - (\bar{x}_1 + \theta_1^*)] \bar{x}_1 x_2. \tag{34}
\]

The desired control input of \( x_2 \) is \( \theta_1^* \) in Eq. (27). Combining Eq. (27) and Eq. (28), we have:

\[
x_{2,des} = \theta_1(\bar{x}_1, \theta_1^*) = -K_r r_{ads,1}(\bar{x}_1(\bar{x}_1 - \theta_1^*)) \frac{\bar{x}_1}{1 - \bar{x}_1^*} \tag{35}
\]

where \( \bar{x}_1 = (x_1 - \theta_1^*) \) and \( 0 < \theta_1^* \leq 1 \). The time derivative of the Lyapunov function is:

\[
\dot{V}_1(\bar{x}_1, t) = -r_{des,1}(t) + r_{red,1}(t) c_{N0,1}(t) + r_{ox1}(t) \bar{x}_1(\bar{x}_1 + \theta_1^*) + r_{ads,1}(t) [1 - (\bar{x}_1 + \theta_1^*)] \bar{x}_1 x_2. \tag{34}
\]

The desired control input of \( x_2 \) is \( \theta_1^* \) in Eq. (27). Combining Eq. (27) and Eq. (28), we have:

\[
x_{2,des} = \theta_1(\bar{x}_1, \theta_1^*) = -K_r r_{ads,1}(\bar{x}_1(\bar{x}_1 - \theta_1^*)) \frac{\bar{x}_1}{1 - \bar{x}_1^*} \tag{35}
\]

which leads to:

\[
\dot{V}_1(\bar{x}_1, t) = -r_{des,1}(t) + r_{red,1}(t) c_{N0,1}(t) + r_{ox1}(t) \bar{x}_1(\bar{x}_1 + \theta_1^*) - \frac{\bar{x}_1}{2 g_1(1 - \theta_1^*)} \frac{\bar{x}_1}{1 - \bar{x}_1^*} \frac{\bar{x}_1(\bar{x}_1 - \theta_1^*)}{(x_4 + \theta_1^*)} + 1] \bar{x}_1^2 r_{ads,1}(t)^2. \tag{36}
\]

Since \( \bar{x}_1 > 0 \), we have:

\[
\left( r_{des,1}(t) + r_{red,1}(t) c_{N0,1}(t) + r_{ox1}(t) \bar{x}_1(\bar{x}_1 + \theta_1^*) \right) > 0, \tag{37}
\]

and

\[
\frac{\bar{x}_1}{2 g_1(1 - \theta_1^*)} \frac{\bar{x}_1}{1 - \bar{x}_1^*} \frac{\bar{x}_1(\bar{x}_1 - \theta_1^*)}{(x_4 + \theta_1^*)} + 1] \bar{x}_1^2 r_{ads,1}(t)^2 \geq 0
\]

because \( \{\bar{x}_1 - \theta_1^* \} \{x_4 - \theta_2^* \} \geq 1 \). So

\[
\dot{V}_1(\bar{x}_1, t) \leq -r_{des,1}(t) + r_{red,1}(t) c_{N0,1}(t) + r_{ox1}(t) \bar{x}_1(\bar{x}_1 + \theta_1^*) \leq -r_{des,1}(t) + r_{red,1}(t) c_{N0,1}(t) \]
where $W_3(\tilde{x}_3, \tilde{x}_2, \tilde{x}_1)$ is continuous positive definite, and the convergence of $x_3$ to $x_{3,\text{des}}$ is guaranteed by Eq. (49).

In summary, based on the Lyapunov functions in Eq. (32), Eq. (40), and Eq. (45), we proved that by the controlling law in Eq. (24), $x_3$ converges to $x_{3,\text{des}}$, $x_2$ converges to $x_{2,\text{des}}$, and also $x_1$ converges to the desired value $\theta_1^*$ disregard of $x_4$.

2) Case 2: $x_1 \leq \theta_1^*$

Considering a Lyapunov function candidate $V_4$ as:

$$V_4(\tilde{x}_4, t) = \frac{1}{2} \tilde{x}_4^2 \geq 0,$$

where $\tilde{x}_4 = x_4 - \theta_1^*$. The time derivative of $V_4$ is:

$$\dot{V}_4 = \tilde{x}_4 \dot{x}_4 = \tilde{x}_4 \left[ f_4(x_3, x_4, \delta_3(t)) + g_4(x_3, \delta_3(t)) x_3 \right],$$

where $f_4 < 0$ and $g_4 > 0$. In order to prove $\dot{V}_4$ to be negative definite, at first, we want to prove the $x_3$ can converge to $\emptyset_2$ in a finite time such that we can assume $x_3 = \emptyset_2$.

A Lyapunov function candidate $V_5$ is selected as:

$$V_5(\tilde{x}_5, t) = \frac{1}{2} \tilde{x}_5^2 \geq 0,$$

following Eq. (45), with the control law in Eq. (24), the time derivative is:

$$\dot{V}_5(\tilde{x}_5, t) \leq \tilde{x}_5 \left[ f_3(x_3, x_3, \delta_3(t)) + g_3(\delta_3(t)) u - \emptyset_2(x_2, x_3, \theta_1^*) \right] \leq K_5 \tilde{x}_5^2 \left/ g_{3,\text{max}} \right.$$

where $W_5$ is positive definite. So we proved that $x_3$ can converge to $\emptyset_2$ in a finite time such that $x_3 = \emptyset_2$ is a valid assumption. And then Eq. (51) becomes:

$$\dot{V}_4 = \tilde{x}_4 \left[ f_4 + g_4 \emptyset_2 \right] = \tilde{x}_4 \left[ f_4 - g_4 + \frac{K_2(x_2 - \theta_1^*) + f_2}{g_2} \emptyset_2 \right],$$

Case 2.1: $x_1 \leq \theta_1^*$, $x_4 \geq \theta_2^*$

In this case, since $x_1 \leq \theta_1^*$ and $x_4 > \theta_2^*$, $\{\text{sign}(x_1 - \theta_1^*)(x_4 - \theta_2^*) + 1\} = 0$ and thus $K_1 = 0$ and $\emptyset_2 = \emptyset_1 = 0$. $V_4$ becomes:

$$\dot{V}_4 = \tilde{x}_4 \left[ f_4 - g_4 + \frac{K_2 x_2 + f_2}{g_2} \emptyset_2 \right],$$

Since $f_4 < 0$, $g_2 > 0$, and $K_2 > \frac{f_{2,\text{max}}}{x_{2,\text{min}}}$, we have:

$$\dot{V}_4 \leq \tilde{x}_4 \left[ f_{4,\text{min}} - g_{4,\text{min}} \frac{K_2 x_{2,\text{min}} + f_{2,\text{min}}}{g_{2,\text{max}}} \right] = -W_6(\tilde{x}_4).$$

where $W_6$ is continuous positive definite, which guarantees that $x_4$ can converge to $\theta_2^*$.

Case 2.2: $x_4 \leq \theta_2^*$, $\tilde{x}_4 < 0$, for $\dot{V}_4$ to be negative definite, according to Eq. (54), we need $f_4 + g_4 \emptyset_2 > 0$.

Combining Eq. (57), Eq. (26), Eq. (27), and Eq. (28), we have:

$$f_4 + g_4 \emptyset_2 = f_4 + \frac{g_4}{g_2} \left[ -K_2(x_2 + K_1) \frac{x_1 - \theta_1^*}{1 - x_1} - f_2 - K_1 \left( \frac{f_1 + g_1 x_2 (1 - \theta_1^*)}{(1 - x_1)^2} \right) \right]$$

Since $x_4 \leq \theta_1^*$ and $x_4 < \theta_2^*$, $K_1 = \frac{c}{g_4(1 - \theta_1^*)}$. Based on Eq. (58), Eq. (57) can be satisfied if:

$$x_1 - \theta_1^* < \frac{g_{2,\text{max}}}{K_2 C} \left( f_{4,\text{min}} - \frac{g_{4,\text{min}} f_{2,\text{ax}}}{g_{2,\text{min}}} \right) - \frac{g_{4,\text{max}}}{C} \frac{- f_{1,\text{min}} + g_{1,\text{max}}}{K_2 C (1 - x_{1,\text{max}})} = -\varepsilon.$$

We can see if $K_2$ and $C$ are chosen large enough, $\varepsilon$ can be very close to zero, which means Eq. (59) can be approximated as $x_1 - \theta_1^* < 0$, which is satisfied in this case. For example, nominal values of $K_2 = C = 100$ leads to $\varepsilon = 0.03$. We conclude here that $x_4$ can converge to the optimal value $\theta_1^*$ if $x_1$ is smaller than $\theta_1^* - \varepsilon$. This is acceptable because $\varepsilon$ can be very small if $K_2$ and $C$ are designed to be large enough. And physically, the $\theta_1^* \geq x_4 > \theta_1^* - \varepsilon$ can be considered as a buffering region which can prevent $x_1$ approaching $\theta_1^*$ too fast which can cause an overshoot and unsatisfied constraint of $x_1 \leq \theta_1^*$.

IV. SIMULATIONS

Figure 2 shows representative simulation results of the proposed controller. The simulations were done based on the first phase of a FTP 75 cycle. The set value of the first cell was set to 0.8 at initial and then step down to 0.3 at 250th second, and the set value of the second cell was 0.3 at initial and step up to 0.5 ad 150th second. As can be seen more clearly in Figure 3, between 0-150 second, the prescribed upper limit of the ammonia reservoir cell, the first cell, is higher than the set value of the NOx conversion cell, the second cell. In this case the coverage ratio of the NOx conversion cell can be controlled to the optimal to provide a most efficient NOx conversion and the ammonia reservoir cell is able to absorb more than expected ammonia slip from the upstream cell. The same situation is in between 150-250 second where the optimal coverage ratio of the NOx conversion cell was raised. In this period the ammonia reservoir cell still possessed a coverage ratio lower than the limit, the coverage ratio of the NOx conversion cell can still be regulated to the optimal value. The prescribed coverage ratio limit of the ammonia reservoir cell was lowered after 250th second. As can be seen in the last plot in Figure 3, in order to prevent unwanted ammonia slip at tailpipe, the coverage ratio of the ammonia reservoir cell is controlled under the prescribed limit and the coverage ratio of the NOx conversion cell was thus forced down to a lower value.

Notice that there are some points, e.g. at time around 170 sec, 200 sec, and 270 sec, the coverage ratios were lowered to the expected values. These are because NOx exhaust from
engine was very high and cannot be compensated even with the maximum NH3 injection.

\[ \theta_{\text{NH3}} \] \text{NH3 surface coverage ratio}
\[ \theta^* \] \text{Optimal surface coverage ratio}
\[ \theta \] \text{NH3 storage capacity of SCR substrate (mole)}
\[ n_{\text{x}} \] \text{Mass flow rate of x into the SCR reactor (g/sec)}
\[ \eta_{\text{NH3}} \] \text{Urea-NH3 conversion efficiency}
\[ C_x \] \text{Mole concentration of species x (mole/m^3)}
\[ r_x \] \text{Reaction rate of species x (mole/sec)}
\[ E \] \text{Activation energy (kJ)}
\[ R \] \text{Gas constant (kJ/kmole/K)}
\[ T \] \text{Temperature (K)}
\[ k_j \] \text{Rate constant of reaction j}
\[ M_x \] \text{Number of mole of species x (mole)}
\[ F \] \text{Exhaust flow rate (m^3/sec)}
\[ v \] \text{SCR reactor volume (m^3)}

V. CONCLUSION

A backstepping based SCR controller is purposed in this paper to control the surface coverage ratios of a two cells SCR model. To minimize the NOx emission and limit the tailpipe ammonia slip under a limit, we want to regulate the coverage ratio of the upstream cell to an optimal value to maximize the NOx conversion efficiency under the condition that the coverage ratio of the downstream cell is below a prescribed value which can ensure a low tailpipe ammonia slip. On the other hand, if the desired coverage ratio of the upstream cell is too high by which the downstream cell coverage ratio can reach over the limitation, the upstream cell coverage ratio can be forced down to prevent undesired ammonia slip. Based on the first phase of a FTP 75 cycle, simulations results verified that the designed controller can regulate the coverage ratios to satisfy the purposed SCR control requirements as expected.

APPENDIX

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